This study presents an approximate two-dimensional inversion procedure for airborne transient electromagnetic data. The method is a two-stage procedure, where data are first inverted with 1D multi-layer models. The 1D model section is then considered as data for the next inversion stage that produces the 2D model section. By assuming translational invariance, the second part of the inversion becomes a deconvolution. The convolution kernels are computed from the 2D sensitivity functions combined with a 1D inversion procedure whereby the time dimension is taken out of the problem and the convolution kernels map directly from the 2D model space to the 1D model space. Within its limitations, the approximate 2D inversion performs quite well. Theoretical modelling shows that it delivers model sections that are a definite improvement over 1D model sections. Comparing 1D inversion of SkyTEM data from a survey line in northern Australia with the approximate 2D inversion, it is seen that model artifacts like pant-legs are identified and removed and conductive anomalies are better resolved.

**Keywords:** TEM, 2D approximate inversion, Deconvolution

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**INTRODUCTION**

1D inversion of airborne TEM data is still a workhorse in the process of going from data collection to interpretation. It is fast, and it delivers good resolution that can be trusted if the underlying conductivity structure is sufficiently 1D, i.e. if lateral gradients in conductivity are not too strong. If not, 1D inversion will produce model artifacts and 2D and 3D inversion becomes necessary. Fortunately such codes are becoming available (and trustworthy) and computation have become tractable for surveys that are not too large (Haber and Schwarzbach 2014; McMillan et al. 2015).

Now, imagine a scale of the dimensionality of the subsurface conductivity from 1D going toward more complex 2D and 3D models. For 1D and almost-1D models, 1D inversion is a satisfactory solution. Then comes an interval where data can still be fitted with 1D models, but where 1D model artifacts occur, and this is the one that is addressed by this study. Beyond that comes the models where 1D models responses will no longer fit the data and 2D or 3D inversion becomes necessary. In the interval where 1D inversion fits the data, but where artifacts might arise, the primary nonlinearity of the inversion problem - the diffusion behaviour - is taken out of the problem, and data as a function of time have actually been transformed to - possibly erroneous - subsurface conductivity as a function of location. What remains is to "push back" the artifacts from where they occur in the 1D model sections to where they belong (the seismic equivalent is migration) characterised by a less complicated functional relationship than the 1D inversion, and it might thus be amenable to an approximate solution.

Previous studies (Wolfgram et al., 2003; Christensen et al., 2006) have shown that an assumption of translational invariance is a good starting point for developing an approximate 2D inversion. This assumption is maintained in this study, but the convolution kernels are now calculated using the 2D sensitivity functions combined with a 1D inversion, whereby the forward mapping maps directly from the 2D model section to the 1D model section. This is a huge advantage and it turns out that it makes the convolution kernels independent on the conductivity of the background halfspace model. Furthermore, the linearity and translational invariance makes it possible to solve the problem as a Multi-Channel Deconvolution problem which speeds up the calculation with several orders of magnitude.

**THEORY**

In the Born approximation, let \( \sigma_0 + \Delta \sigma_{2D} \) be an anomalous element of the 2D model section. Using the approach of Christensen (2014) to derive the 2D sensitivity function, \( F_{2D} (\cdot) \), the mapping into time domain apparent conductivity is given by:

\[
\sigma_a (\sigma_0, t, x') = \sigma_0 + \int_0^\infty dz \int_{-\infty}^{\infty} \sigma_{2D}(x, z) \
\times F_{2D}(\sigma_0, t, x' - x, z) \, dx
\]  

(1)

The discrete 1D problem for \( \sigma_a \) is given as (Christensen 2016):

\[
\sigma_a = G \cdot \sigma_{1D}
\]  

(2)
\[ G_{i,j} = \int_{z_j}^{z_{j+1}} F_{1D}(\sigma_0, t_i, z) \, dz \]

\[ F_{1D}(\sigma_0, t, z) = \int F_{2D}(\sigma_0, t, x, z) \, dx \]  \hfill (3)

\( G \) is the matrix of 1D sensitivity integrated over the 1D model layers. The least squares inversion form of (2) is:

\[ \sigma_{1D}(z) = (G^T G)^{-1} G^T \cdot \sigma_a(t) \]  \hfill (4)

where it is assumed that all data errors are the same for all \( \sigma_a \) values, so the data error covariance matrix is omitted.

Introducing the differential 1D conductivity \( \Delta \sigma_{1D}(z) \) we have:

\[ \sigma_0 + \Delta \sigma_{1D}(z) = (G^T G)^{-1} G^T \cdot (\sigma_0 + \Delta \sigma_{2D} \cdot W(t)) \]  \hfill (5)

where \( W \) is the integral of the 2D Fréchet kernel over a 2D model elements and \( \Delta \sigma_{2D} \) is the perturbation of a 2D model element. If there is no anomalous conductivity, \( \Delta \sigma_{2D} = 0 \), then the resulting 1D conductivity must be equal to \( \sigma_0 \), so

\[ \sigma_0 = (G^T G)^{-1} G^T \cdot \sigma_0 \]  \hfill (6)

which shows that the sums of the rows of \( (G^T G)^{-1} G^T \) must be unity. Subtracting (6) from (5) we have:

\[ \Delta \sigma_{1D}(z) / \Delta \sigma_{2D} = (G^T G)^{-1} G^T \cdot W(t) \]  \hfill (7)

where the ratio \( \Delta \sigma_{1D}(z) / \Delta \sigma_{2D} \) establishes the convolution kernel. In practical calculations, the inversion will have to be regularized so the inversion problem is in fact formulated as:

\[ \sigma_{1D}(z) = (G^T G + \frac{1}{\text{var}_C} C_{\sigma}^{-1})^{-1} G^T \cdot \sigma_a(t) \]  \hfill (8)

where \( C_{\sigma} \) is the model covariance matrix imposing the vertical smoothness in the 1D inversion with the variance \( \text{var}_C \). The regularisation term is chosen pragmatically so that it is as small as possible while still producing a regularisation that will ensure that the convolution kernels are not erratic and that the 1D inverse mapping produces the best inversion models, i.e. not too smooth and not too oscillatory.

In the above derivations, the end result - the 2D convolution kernel - is not a function of time, but delay times in a proper time interval should be used in the derivations to make sure that the kernels are constructed in the best way. AEM data are recorded at different heights so convolution kernels for different heights are needed.

Figure 1. The convolution kernels depicting the contribution of a 2D model cell (marked as a black rectangle) to the anomalous 1D conductivity model section for the 2D anomaly in layer 5, 15, and 25. The inverse kernel for layer 20 is seen in the bottom frame.
Even though the background conductivity, $\sigma_0$, has been eliminated from equation (7), the elements of the Jacobian, $G$, still depend on $\sigma_0$. However, it turns out that eventually the convolution kernels will not depend on the background conductivity. The convolution kernels for three different depths of the 2D anomaly are seen in Figure 1. The 2D and 1D model sections are discretised laterally with cells of 10 m width from -1500 m to 1500 m and vertically with 30 layers where the depths to the layer boundaries increase as a sinh function. The top layer thickness is 2 m and the bottom layer boundary is at 500 m.

The similarity between the convolution kernels of Figure 1 with the corresponding seismic migration convolution kernels makes it interesting to see if this similarity is maintained when the ‘inverse’ kernels are considered. The inverse kernel depicts the relative contributions of 2D model cells to a single 1D model cell. In seismics, these kernels are called the ‘Smiles’. Figure 2 shows the inverse kernel for the TEM problem for a 1D model cell in the 20th layer, and it is seen that the similarity with the seismic situation is maintained.

**FORWARD AND INVERSE MODELLING**

Given the convolution kernels, the Jacobian, $G$, of the forward modelling can be constructed and the inversion formulated. The forward modelling is simply given as:

$$ m_{1D} = G \cdot m_{2D} $$

and the inverse problem is formulated as a straightforward least-squares inversion:

$$ m_{2D} = C_m G^T \cdot (C_c + G C_m G^T)^{-1} \cdot m_{1D} $$

where the alternative inversion formulation of equation (41) in Tarantola and Valette (1982) has been used. As model covariance matrix, $C_m$, a broadband von Karman covariance function is used (Serban and Jacobsen 2001; Christensen et al. 2009, Maurer et al. 1998). Equation 10 is the straightforward inversion formulation, and in these initial investigations no attempts are made to make the inversion more effective.

First I will present some internal checks of the convolution/deconvolution approach by forward calculating the response of simple models, add random noise with an effective amplitude of 3 mS/m to the response to produce the theoretical data and then invert these data. Figure 2 shows the true model, the forward response without noise, and the inverted 2D model. As expected, the forward response exhibits the well-known pant-leg structures, but the inversion reproduced a version of the true model that is more fuzzy.

**Figure 2.** Top: The theoretical true model is a 50 x 100 m block with conductivity 0.10 S/m in a homogeneous halfspace of 0.05 S/m. Middle: The forward response, i.e. the 1D model section. Bottom: the inverted (deconvolved) model. The block outline is indicated with a black line.

**Figure 3.** Top: The theoretical true model is two dipping dykes with conductivity 0.10 S/m in a homogeneous halfspace of 0.05 S/m separated by 80 m. Middle: The forward response, i.e. the 1D model section including the added noise. Bottom two frames: the inverted (deconvolved) model; the top one is for a dyke separation of 80 m; the bottom one for a separation of 120 m.
due to the regularization, but otherwise capture the true model well.

Next I will look at two dykes dipping ~45° and with separations 80 m and 120 m. The results are shown in Figures 3. In none of the 1D model response sections is it obvious that there are two distinct dykes, but the 2D inversion separates the anomalies again - as one would wish - clearly so in for the 120 m separation, less clearly for the 80 m separation.

Figure 4 shows the results of applying the 2D deconvolution to field data. The figure shows a model section of 1D inversions of a line flown with the SkyTEM system in Northern Australia and the result of the 2D deconvolution. It is seen that some of the pant-leg-like structures at the near-surface are removed and that the lateral smearing of the conductive anomalies is reduced. It should be remembered that the structures appearing at the ends of the line are only covered by data from one side and they are therefore more uncertain.

![Figure 4. Top: Model section of 1D inversion of SkyTEM data from a line in Northern Australia. Bottom: The results after 2D deconvolution.](image)

**CONCLUSIONS**

In the model complexity range where transient AEM data can be fitted with 1D inversion models, but where model artifacts may arise because the underlying conductivity structure is not 1D, this study suggests a correction procedure based on a convolution/deconvolution approach. Convolution kernels are derived that directly maps from a 2D model section to a 1D model section and solving the inversion problem recovers the 2D model within the resolution capabilities of the method. Internal check in the form of forward modelling of theoretical models shows that the expected artifacts are reproduced and subsequent inversion shows that the suggested procedure has internal consistency and recovers the 2D model. Application to field data suggests that the occurrence of model artifacts is reduced.

**ACKNOWLEDGMENTS**

Parts of this work was supported by the Groundwater Branch at Geoscience Australia and funded by GA as part of a collaboration project between GA and Aarhus University.

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