MATLAB based code for 3D joint inversion of Magnetotelluric and Direct Current resistivity imaging data

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**SUMMARY**

A new code, AP3DMT\_DC, for joint inversion of 3D Radio Magnetotelluric (RMT) and Direct Current Resistivity (DCR) data is developed and tested over synthetic model. The code is an extension of our recently developed MATLAB based code (AP3DMT) developed for 3D inversion of MT data. Motivation for this problem came from the fact that the numerical solution of a DCR problem in AP3DMT is an integral part of MT problem. 3D DCR problem has been solved in implementing divergence correction numerical solution of 3D MT problem. In the present paper we extended AP3DMT code for the solution of DCR problem individually and jointly and named the new code as AP3DMT\_DC. In this code we have implemented 3D DCR inversion independently and jointly with MT. As the sensitivities of MT and DCR data are different for resistive and conductive structures, therefore, the joint inversion of MT and DCR data has strengthened the weakness of individual method. We have found through synthetic examples that by using optimally discretization, model scaling and error floor joint inversion produced better resolved model for conducting and resistive features in comparison to the individual inversion. The performance of the code has been demonstrated over a synthetic example.

**Keywords:** DCR inversion, 3D joint inversion of MT and DCR, Divergence correction

**INTRODUCTION**

MT and DCR methods are normally used to determine the subsurface conductivity/ resistivity distribution from the limited set of measurements carried over the earth's surface. However, physical principal and thus sensitivities of these methods are different for conductive and/or resistive structure. Thus it is expected that the joint inversion will improve the resolution of the model for conductive and resistive structure where individual method are generating poorly resolved model. The joint inversion may be performed either by inverting independently each data set and integrating the results manually or two different data sets are inverted together to constrain the interpretation (Bastani et al. 2012; Candansayar and Tezkan 2008; Kalscheuer et al. 2013; Seher and Tezkan 2007b; Sasaki 1989). 2D joint inversion were performed by Sasaki (Sasaki 1989) followed by various authors (e.g. Candansayar and Tezkan 2008; Seher and Tezkan 2007b; Shan et al. 2014). Recently, (Amatyakulet al. 2017) presented WSJointInv 2D-MT-DCR a joint inversion program for 2D MT and DCR data based on Occam’s data space technique. If the data is of 3D nature, 2D inversion algorithm may not be an accurate choice (Sasaki 1989). In such situation 3D inversion is a better choice. We have recently developed 3D MT inversion code (Singh et al. 2017) and also implementing divergence corrections in 3D MT computation. Thus in divergence correction we have solved DCR problem. We are motivated to extend AP3DMT code for the solution of DCR problem individually and jointly.

**JOINT INVERSION OF MT AND DCR DATA**

We have extended AP3DMT (Singh et al. 2017) code to the implement individual and joint inversion of MT and DCR data and rename it as AP3DMT-DC code. The assemblage of data sets of MT and DCR and Jacobian are discussed here. Three crucial aspects, model discretization, data scaling and error floor, are crucial aspect and need to be done optimally. Finally individual and joint inversion of one synthetic test example is presented.

The penalty function for the joint inversion of MT and DCR data after transformation can be written as:

\[
\varphi(m, d^{obs}) = (d^{obs} - F(m))\top(d^{obs} - F(m)) + \lambda m\top m
\]  

(1)

Where \(d^{obs}\) are MT, DCR data, \(F(m)\) is the forward response of model \(m\) and \(\lambda\) is the regularization parameter. For implementation of joint inversion, different data sets of MT and DCR must be combined as

\[
\begin{bmatrix}
    d_{MT} \\
    d_{DCR}
\end{bmatrix} = 
\begin{bmatrix}
    F_{MT}(m) \\
    F_{DCR}(m)
\end{bmatrix};

\begin{bmatrix}
    d_{MT}^{obs} \\
    d_{DCR}^{obs}
\end{bmatrix} = 
\begin{bmatrix}
    d_{MT}^{obs} \\
    d_{DCR}^{obs}
\end{bmatrix}
\]

(2)
with, DCR apparent resistivity, $d_{DCR}^{app}$ = $[\rho^2]$ and magnetotelluric data, $d_{MT}^{app}$, could be impedance tensor or apparent resistivity and phase response defined as:

$$d_{MT}^{app} = \begin{bmatrix} Z_{xx}^{MT} & Z_{xy}^{MT} & Z_{yx}^{MT} & Z_{yy}^{MT} \\ Z_{yx}^{MT} & Z_{yy}^{MT} & Z_{yx}^{MT} & Z_{xx}^{MT} \\ Z_{xy}^{MT} & Z_{yx}^{MT} & Z_{xx}^{MT} & Z_{xy}^{MT} \\ Z_{yy}^{MT} & Z_{yx}^{MT} & Z_{xy}^{MT} & Z_{yy}^{MT} \end{bmatrix}_{OT}$$

(3)

$$C_d = \begin{bmatrix} C_{MT}^{NF} & 0 \\ 0 & C_{DCR}^{NF} \end{bmatrix}$$

(4)

The total number of the data $N_d$ for the joint inversion is the sum of the number of both data types, $N_d = N_{MT} + N_{DCR}$.

The data covariance matrix, diagonal matrix, is also written such that its size is same as the number of both data types written as,

$$C_d = \begin{bmatrix} C_{MT}^{NF} & 0 \\ 0 & C_{DCR}^{NF} \end{bmatrix}$$

(5)

Special care must be taken for model discretization, data scaling, error floor etc.

**EXAMPLE OF MT AND DCR JOINT INVERSION**

We have selected modified version of 3D checkerboard pattern model (Egbert and Kelbert 2012) to demonstrate the performance of 3D MT and DCR joint inversion. The model consists of a 100 m thick top layer with resistivity 100 Ω-m and bottom layer resistivity of 10 Ω-m. The top layer, Figure 1, consists of nine conductive and nine resistive blocks (10 and 1000 Ω-m respectively). The top nine blocks are lying in the depth between 0−20 m while lower 9 blocks are between 40−70 m depths. These two sections (0−20 m and 40−70 m) are named as L1 and L2 and are likely sensed by both the method (MT and DCR). For synthetic data generation, the model was discretized into 61 × 71 × 23 cells (excluding cells in air for MT case), with nominal resolution of 10 m in horizontal direction. For the DCR data, 10 profiles were placed, with inter-profile spacing 50−60 m, covering a length of 480 m (Figure 1). In each profile DCR data was generated for 31 electrodes with a spacing of 20 m. Apparent resistivity data were computed for dipole-dipole configuration with dipole length of 20 m and separation factor $n = 1−10$. For MT data, stations were placed 40 m apart between ±240 m on each profile (130 total stations). Data (off diagonal impedance tensor) was generated for 10 periods, logarithmically spaced between $10^{-3}−10^{-1}$ s. The combined data set has 5200 and 2350 data points for MT and DCR respectively. Two percent random noise was added to both data. The error floor for MT and DCR were then set at 2% of the absolute values of the responses. For all the inversion runs, a homogeneous model of 100 Ω-m, was used as apriori and initial model. The initial model was discretized into same number of cells as the synthetic model.

**Figure 1.** (a) Plan view of resistivity model of 3D test, with circles (red) representing electrodes and circles (blue) representing MT station, and (b) cross −section view of the model at x=0 profile.

Three inversions were performed. First we invert the MT data only. In 51 $NLCG$ iterations the nRMS error reduced from 58.4 to 1.24. The inverted model thus obtained is shown in Figures 2b. Next, we invert the DCR data only. In 34 $NLCG$ iterations the nRMS error reduced from 101.5 to 1.22 resulting in inverted model shown in Figure 3a. Finally, MT and DCR data set were inverted jointly. Before inversion, the DCR data set was up-weighted based on the number of data points in each set. In 52 $NLCG$ iterations the nRMS error reduced from 97.2 to 1.45.
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The inverted model thus obtained is shown in Figure 3b. The inverted model for MT inversion, DCR inversion and MT-DCR joint inversion will be called as M1, M2 and M3 respectively.

Figure 2: (a) Resistivity model used to generate synthetic data for 3D MT test, and (b) Inverse model obtained after 51 NLCG iterations with MT data only.

Figure 3. (a) Inverse model obtained after 34 NLCG iterations with DCR data only, and (b) Joint inverse model obtained after 52 NLCG iterations with MT and DCR data. Note that in the cut-away view the upper surface shown is at 10 m depth, but the structures shown extend to the surface.

In terms of imaging capability for shallow conductive features MT and DCR gives same results while for resistive features DCR gives better result as compared to MT. For the shallow features there seems to be no superiority of joint inversion over individual inversion as DCR inversion alone gives result similar to MT-DCR inversion.

CONCLUSIONS

A new MATLAB based code, referred as AP3DMT_DC, for the individual or joint inversion of MT and DCR data is developed and tested over a synthetic model. Through synthetic experiment it is demonstrated that joint inversion results in better inverse models as compared to individual inversion. It is observed that data scaling proves to be very crucial. Here, we have presented synthetic examples case. However, the code need to extensively tested and validated over a variety of field examples in future.

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REFERENCES


